

# Measuring Fiscal Impoverishment

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# Motivation: Debate about taxing the poor

- Governments criticized for heavily taxing the poor

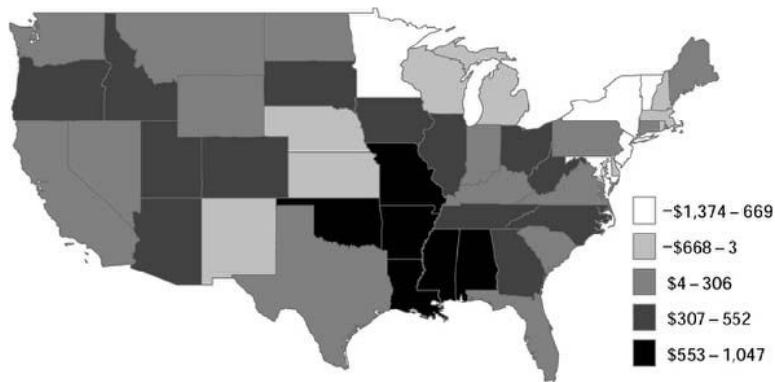
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State and local tax burden for a family of three at the poverty line



Source: [Newmann and O'Brien \(2011\)](#)

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- Example: southern states of US
  - Letter from Louisiana clergy to Governor Jindal:

“We are concerned that Louisiana already has one of the most regressive tax systems in the nation, putting a disproportionately high burden on low income families. [. . .] That is unacceptable.”

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    - ▶ Politicians (e.g., Rodrigues, 2011)
    - ▶ Academics (Siqueira and Nogueira, 2013)
    - ▶ Multilateral organizations (Afonso et al., 2013)
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- World Bank recommendation to developing countries: “avoid taxing the poor”



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- Current measures of tax and transfer system inadequate

# Brazil

		Post-tax and transfer income groups				% of Pop.
		< \$2.50	\$2.50 -4.00	\$4.00 -10.00	> \$10.00	
Pre-tax and transfer income groups	< \$2.50	85%	10%	4%	1%	15%
	\$2.50 -4.00	14%	75%	10%	1%	11%
	\$4.00 -10.00	0%	13%	84%	3%	33%
	> \$10.00	0%	0%	16%	84%	40%
% of Pop.		14%	14%	36%	36%	100%

1. Show that standard measures of the effect of taxes and benefits on the poor
  - Poverty indicators (including squared poverty gap)
  - Stochastic dominance tests
  - Measures of horizontal inequity and progressivitydo not tell us whether some of the poor are made poorer by the tax and transfer system (“fiscal impoverishment”)

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2. Illustrate that this phenomenon is occurring in Brazil
3. Axiomatically derive measures that *do* capture FI
  - FI headcount, gap, and log gap
  - FI curve and dominance criteria



# Defining Fiscal Impoverishment

- Income space  $\Omega \subset \mathbb{R}_+$  and  $\sup \Omega < \infty$
- Income before taxes and transfers  $y_i^0 \in \Omega$  and after taxes and transfers  $y_i^1 \in \Omega$  for  $i = 1, \dots, n$
- Cumulative distribution functions  $F_0 : \Omega \rightarrow [0, 1]$  and  $F_1 : \Omega \rightarrow [0, 1]$
- Poverty line  $z \in \Omega$
- There is **fiscal impoverishment** if  $y_i^1 < y_i^0$  and  $y_i^1 < z$  for some  $i$

# Review of Stochastic Dominance

- Let  $F$  and  $G$  be the cumulative distribution functions for two income distributions.
- $F$  (weakly) first order stochastic dominates  $G$

$$\text{if } F(y) \leq G(y) \quad \forall y$$

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- $F$  first order stochastically dominates  $G$  on  $[0, z]$ 
  - ⇔ Lower poverty under distribution  $F$  for broad class of poverty measures, any poverty line  
(Atkinson 1987; Foster and Shorroks 1988)

# Review of Horizontal Inequity and Progressivity

- **Horizontal inequity** occurs when pre-tax and transfer equals are treated *unequally* by the fiscal system
  - or individuals are reranked by the fiscal system
- There is **classical horizontal inequity** if  $y_i^0 = y_j^0$  and  $y_i^1 \neq y_j^1$  for some  $(i, j)$  pair
- There is **reranking** if  $y_i^0 \geq y_j^0$  and  $y_i^1 < y_j^1$  for some  $(i, j)$  pair

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- There is **reranking** if  $y_i^0 \geq y_j^0$  and  $y_i^1 < y_j^1$  for some  $(i, j)$  pair
- The tax and transfer system is **progressive** if net taxes—i.e., taxes minus benefits—as a proportion of income increase with income

# Propositions: FI and FOSD

- $F_1$  does not weakly FOSD  $F_0$  among the poor
  
  
  
  
  
  
  
  
  
  
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 $F_1$  FOSD  $F_0$  on  $[0, z] \Leftrightarrow$  no FI*

- and there is reranking among the poor

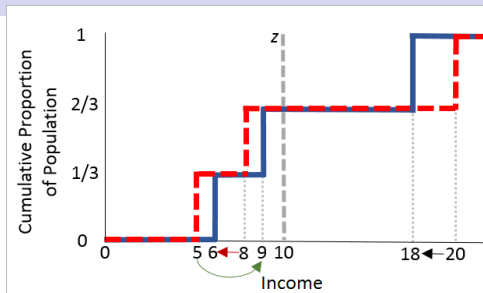
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## Proposition

If there is reranking among the poor,  $F_1$  FOSD  $F_0$  on  $[0, z]$  is **not a sufficient condition** for no FI

## Proof.

$y^0 = (5, 8, 20)$ ,  $y^1 = (9, 6, 18)$ ,  $z = 10$ .  $F_1$  FOSD  $F_0$  among the poor and there is FI □



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Horizontal inequity (classical and reranking) has occurred but FI has not.



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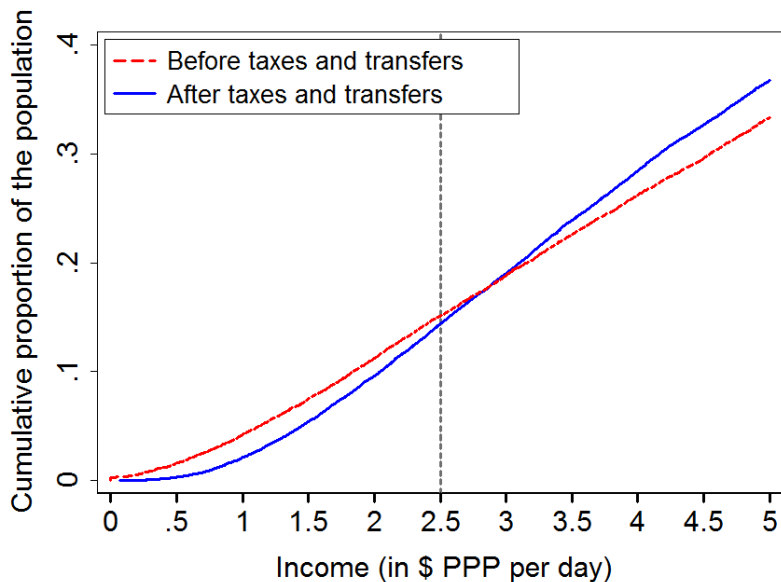
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## Proposition

A globally progressive tax and transfer system is **neither a necessary nor sufficient condition** for no FI.

# An Illustration: Brazil





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- FI gap

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- FI log gap (restrict  $\Omega \subset \mathbb{R}_{++}$ )

$$\ell(\mathbf{y}^0, \mathbf{y}^1) = n^{-1} \sum_{i=1}^n (\ln(\min\{y_i^0, z\}) - \ln(\min\{y_i^0, y_i^1, z\}))$$

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- $h(\mathbf{y}^0, \mathbf{y}^1) = 5.3\%$  of total population
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  - ⇒ If policy could be perfectly targeted to those who are impoverished, its elimination would not be particularly costly
- However, average amount an impoverished person is impoverished =  $g(\mathbf{y}^0, \mathbf{y}^1)/h(\mathbf{y}^0, \mathbf{y}^1) = \$0.19$  per day
  - 10% of their income on average

# Measuring FI

- FI headcount ratio, gap, and log gap can be sensitive to choice of poverty line  $z$
- FI curve
  - For a given cut-off, proportion of total population that was fiscally impoverished

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- Downward mobility curve (Foster and Rothbaum, 2013)
  - For a given cut-off, proportion of total population that experiences downward mobility across that cut-off

$$m(z, \cdot) = n^{-1} \sum_{i=1}^n \mathbb{1}(y_i^1 < z < y_i^0)$$

# Dominance Propositions

- Comparing two post-fisc situations A and B with same pre-fisc distribution
  - e.g., actual situation vs. proposed reform

## Proposition

*A has an unambiguously lower FI headcount ratio than B for all poverty lines  $z \in [z^-, z^+]$   $\Leftrightarrow$   
FI curve of A first order dominates that of B on  $[z^-, z^+]$*

## Proposition

*A has unambiguously lower FI gap than B  $\Leftrightarrow$   
downward mobility curve of A second order dominates that of B on  $[0, z^+]$*