Measuring Impoverishment: An Overlooked Dimension of Fiscal Incidence

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Overview

- Standard measures of the effect of taxes and benefits on the poor
  - Poverty indicators (including squared poverty gap)
  - Inequality indicators
  - Stochastic dominance tests
  - Lorenz dominance tests
  - Measures of progressivity
  - Vertical and horizontal inequity
  - Anonymous social welfare comparisons

...leave out important information about how the poor are affected by fiscal policy.
Overview

• For example, we can have:
  – First (and higher) order stochastic dominance
    ⇒ poverty (including the squared poverty gap) declining
    ⇒ generalized Lorenz dominance
  – Income distribution becoming less unequal
  – Progressive net taxes
  – Low or no horizontal inequity
  – But some of the poor become substantially poorer

• Extent to which a tax and transfer system impoverishes poor or makes non-poor people poor is valuable information for analyst and policymaker

• Formally define impoverishment and establish its relationship with traditional measures

• Propose using a Markovian transition matrix and an income loss matrix
Definitions

• Well-being space $\Omega$
  
  – Could be income ($\Omega \subset \mathbb{R}_+ \text{ and } \sup \Omega < \infty$) or multi-dimensional

• Income before taxes and transfers $y^0_i \in \Omega$ and after taxes and transfers $y^1_i \in \Omega$

• Cumulative distribution functions $F_0 : \Omega \rightarrow [0, 1]$ and $F_1 : \Omega \rightarrow [0, 1]$

• Poverty line $z \in \Omega$

• There is **impoverishment** if $y^1_i < y^0_i$ and $y^1_i < z$ for some $i$
First Order Dominance

- Post-tax and transfer distribution does *not* weakly FOSD pre-tax and transfer distribution among the poor ⇒ *impoverishment*
  - Proof: see paper.

- Post-tax distribution *does* weakly FOSD pre-tax distribution among the poor and tax-benefit system rank-preserving ⇒ *no impoverishment*
  - Proof: see paper.

- Post-tax distribution *does* weakly FOSD pre-tax distribution among the poor and re-ranking ⇒ dominance cannot determine impoverishment
  - Proof: $y_0 = (5, 8, 20), y_1 = (9, 6, 18), z = 10$. $F_1$ FOSD $F_0$ among the poor and there is impoverishment.
Horizontal Equity and Progressivity

• Horizontal inequity is neither a necessary nor sufficient condition for impoverishment.
  – Not sufficient:
    \( y^0 = (5, 5, 6, 20), y^1 = (5, 7, 6, 18), z = 10 \). Horizontal inequity (classical and re-ranking) has occurred but impoverishment has not.
  – Not necessary:
    \( y^0 = (5, 8, 20), y^1 = (6, 7, 20), z = 10 \). Impoverishment has occurred but horizontal inequity (classical or re-ranking) has not.

• A progressive tax-benefit system is neither a necessary nor sufficient condition for no impoverishment.
  – Proof: see paper.
Fiscal Mobility Matrix

- Directional mobility literature provides a useful framework
  - See, for example, Fields (2008)
- Compare the status of identified individuals in the before and after taxes and transfers situations
- One can see which individuals are adversely/favorably impacted by a particular policy
- We establish dominance criteria so that alternative policies can be compared in terms of the downward mobility they induce
Definitions

- **Fiscal Mobility**
  - The directional movement between the before and after net taxes situations among $k$ pre-defined income categories

- **Fiscal Mobility Matrix**
  - $k \times k$ transition matrix $P$ where the $ij$-th element $p_{ij}$ is the probability of moving to income group $j$ after net taxes for an individual in group $i$ before net taxes

  $$\Rightarrow P \text{ is a stochastic matrix with } \sum_{j=1}^{k} p_{ij} = 1 \text{ } \forall i \in \{1, \ldots, k\}$$

- **Poverty Lines**
  - Let $z$ be a vector of poverty lines between $\bar{z}$ and $z$. These poverty lines determine a subset $r$ of the $k$ income categories ($r < k$) that are considered poor
Downward Mobility Among the Poor

- If $\sum_{i=1}^{r} \sum_{j<i} p_{ij} > 0$, then there is downward mobility among the poor
- If $\sum_{i=r+1}^{k} \sum_{j<r} p_{ij} > 0$ then there is downward mobility of some non-poor into poverty
  - Example: $k = 6$ and $r = 3$
An Illustration: Brazil

- Income distribution after taxes and transfers
  Lorenz dominates distribution before taxes and transfers
  \[ \Rightarrow \text{Inequality unambiguously falls} \]
  - To illustrate: Gini falls from 0.57 to 0.54
An Illustration: Brazil

- First order stochastic dominance over domain of extreme poverty lines ($\leq 2.50$ PPP per day)
- Second order stochastic dominance over domain of poverty lines ($\leq 4.00$ PPP per day)
An Illustration: Brazil

- Progressive overall tax system:
  - Kakwani index of direct and indirect taxes is 0.04
  - Kakwani index of direct transfers is 0.54
  - Reynolds-Smolensky index of after taxes and transfers income with respect to before taxes and transfers income is 0.05

- However:
  - 11.4% of the moderate poor become extreme poor
  - 10.5% of the vulnerable become poor
## Fiscal Mobility Matrix: Brazil

<table>
<thead>
<tr>
<th>Pre-tax and transfer income groups</th>
<th>Post-tax and transfer income groups</th>
<th>% of Pop.</th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2.50</td>
<td>&lt; 2.50</td>
<td>15.4%</td>
<td>$1.45</td>
</tr>
<tr>
<td>2.50–4.00</td>
<td>2.50–4.00</td>
<td>11.3%</td>
<td>$3.24</td>
</tr>
<tr>
<td>4.00–10.00</td>
<td>4.00–10.00</td>
<td>33.5%</td>
<td>$6.67</td>
</tr>
<tr>
<td>&gt; 10.00</td>
<td>&gt; 10.00</td>
<td>39.8%</td>
<td>$28.41</td>
</tr>
</tbody>
</table>

### Pre-tax and transfer income groups
- < 2.50
- 2.50–4.00
- 4.00–10.00
- > 10.00

### Post-tax and transfer income groups
- < 2.50
- 2.50–4.00
- 4.00–10.00
- > 10.00

### % of Mean
- < 2.50
- 2.50
- 4.00
- > 10.00
How Much do the Losing Poor Lose?

- Matrix of average proportional losses
  - $k \times k$ matrix $L$ with $ij$-th element $\ell_{ij}$ equal to the average percent decrease in income of those who began in group $i$ and lost income due to taxes and transfers, ending in group $j \leq i$
  - Negative semi-definite and weakly lower-triangular by construction
  - There is impoverishment among the poor if and only if $\ell_{ij} < 0$ for some $j \leq r$
## Average Proportional Losses: Brazil

<table>
<thead>
<tr>
<th>Pre-tax and transfer income groups</th>
<th>&lt; 2.50</th>
<th>2.50–4.00</th>
<th>4.00–10.00</th>
<th>&gt; 10.00</th>
<th>% of Pop.</th>
<th>Group Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2.50</td>
<td>−10% $1.93</td>
<td></td>
<td></td>
<td></td>
<td>15.4%</td>
<td>−10% $1.93</td>
</tr>
<tr>
<td>2.50–4.00</td>
<td>−17% $2.72</td>
<td>−11% $3.38</td>
<td></td>
<td></td>
<td>11.3%</td>
<td>−12% $3.28</td>
</tr>
<tr>
<td>4.00–10.00</td>
<td>−18% $4.37</td>
<td>−16% $7.03</td>
<td></td>
<td></td>
<td>33.5%</td>
<td>−16% $6.70</td>
</tr>
<tr>
<td>&gt; 10.00</td>
<td></td>
<td></td>
<td>−21% $11.02</td>
<td>−21% $31.80</td>
<td>39.8%</td>
<td>−21% $28.85</td>
</tr>
</tbody>
</table>

% of Pop.: 14.3% 13.9% 36.0% 35.8% 100%
Fiscal Mobility Dominance

- In terms of fiscal mobility, is an alternative scenario more desirable for the poor than the actual scenario?
- Compare two fiscal mobility matrices $P$ and $P'$ and denote strong downward mobility dominance by the binary relation $\mathcal{M}$
  
  - $P \succ M P'$ if $P$ exhibits less downward mobility among the poor (and into poverty) than $P'$
  
  - Formally, $P \succ M P'$ if $\sum_{m=1}^{j} p_{im} \leq \sum_{m=1}^{j} p'_{im}$ for $i \in \{2, \ldots, k\}$ and $j \leq r < i$, with strict inequality for some $i$
Alternative Scenario: Neutral Tax

- Compare actual scenario in Brazil to an alternative
- Neutral (horizontally equitable) tax
  - Individuals are taxed proportional to their incomes such that total tax revenue remains fixed
- Transfers received are still as observed
- *Ex ante*, difficult to determine whether neutral tax system will entail more or less impoverishment than actual tax system
- 16% of moderate poor become extreme poor
- 15% of vulnerable become moderately poor
## Fiscal Mobility Matrix: Neutral Tax

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<thead>
<tr>
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</tr>
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<td>&gt; 10.00</td>
<td>&gt; 10.00</td>
<td>39.8%</td>
<td>$28.41</td>
</tr>
<tr>
<td>% of Pop.</td>
<td></td>
<td>100%</td>
<td>$14.14</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th></th>
<th>&lt; 2.50</th>
<th>2.50–4.00</th>
<th>4.00–10.00</th>
<th>&gt; 10.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2.50</td>
<td>85%</td>
<td>10%</td>
<td>4%</td>
<td>1%</td>
</tr>
<tr>
<td>2.50–4.00</td>
<td>16%</td>
<td>73%</td>
<td>10%</td>
<td>1%</td>
</tr>
<tr>
<td>4.00–10.00</td>
<td>0%</td>
<td>15%</td>
<td>82%</td>
<td>3%</td>
</tr>
<tr>
<td>&gt; 10.00</td>
<td>0%</td>
<td>0%</td>
<td>17%</td>
<td>84%</td>
</tr>
</tbody>
</table>
Bourguignon’s Welfare Dominance
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